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## Transmission through a mesoscopic ring with a quantum dot

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### Abstract

The transmission through a mesoscopic ring with a quantum dot embedded in one of its arms is studied with a one-dimensional scattering model. The quantum dot is approached by a quantum well scatterer. With the use of a scattering matrix describing the junctions between the leads and the ring, it is analytically shown that the quantum interference and the resonant tunneling dominate the transmission. When the state of the dot is far from a resonance, the system acts as a quantum wire with two separated stubs at both ends. However, when a resonant tunneling through the dot occurs, an extra phase shift may be introduced to the wave through the dot and then the quantum interference effect may be flipped. The dependence of the total transmission coefficient on the properties of the quantum dot is also presented. © 1999 Elsevier Science B.V. All rights reserved.

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It has been well known that quantum interference effects are present in mesoscopic systems, where the wave nature of electrons plays an important role. To characterize the transmission properties fully, the phase information of electrons through such a system is required. Recently, the phase behaviors of electrons traversing through a quantum dot (QD) were extensively studied both experimentally and theoretically [1–7].

In general, the measurements of phase are accomplished by some interferometry systems [1–3]. With the help of Aharonov–Bohm (AB) effects, the first

experimental study on the transmission phase through a QD in a mesoscopic ring was accomplished by Yacoby et al. in 1995 [1]. They observed two features in this experiment: first, the phase of AB oscillations changes abruptly by  $\pi$  when the conductance of the AB ring reaches a peak; second, the AB oscillations at consecutive conductance peaks are in phase. To explain these features, previous studies have been mainly focused on the electron–electron interaction in the QD [4,5], since it is obvious that the phase difference between two consequent resonant states is  $\pi$  in non-interacting models, where the experimental results seem inexplicable. However, by considering both the quantum interference effects and the resonant tunneling through the QD, we showed the phase features of conductance peaks,

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which agree well with the experiment, in a non-interacting model [7]. But, it is still valuable to reveal how the quantum interference and resonance of the QD affect the total transmission properties in this system in a more physical way.

It is the purpose of this paper to present a clear physical picture of the transmission for such a system within a one-dimensional scattering model. As shown in Fig. 1, the system is modeled as a ring attached by two ideal leads from both sides. The QD is characterized as a scatterer by a quantum well (well potential  $-V_G$ ) with two barriers (barrier heights  $V_B$ ) [6,7]. The effective electrostatic potential outside the dot is referred to as  $V_0$ , and is set as zero in the following discussion. Because the single-channel case provide a good approximation to a real wire with finite width at low temperatures, we restrict ourselves to the one-channel case throughout our analysis.

As the first step, we study a ring with arbitrary scatterer in both arms. The scatterer are parameterized by  $t_i$  ( $i = 1, 2$ ) and  $t'_i$ , the transmission coefficient

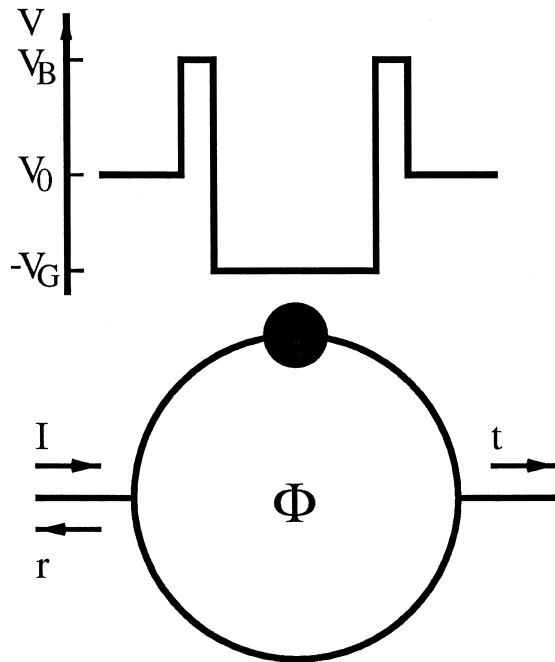


Fig. 1. Geometry of a mesoscopic ring with a quantum dot embedded in one of its arms. The potential well model of the quantum dot is shown at the top.

TC) from the left and from the right, respectively, and  $r_i$  ( $r'_i$ ), the reflection coefficient (RC) on the left (right) of the scatterer, where  $i = 1$  (2) denotes the scatter in the upper (lower) arm. The current-conservation and time-reversal invariance [8,9] imply  $t_i = t'_i$  and  $-t_i/t'_i = r_i/r'_i$ .

Following previous authors [9–11], we describe each junction between the ring and the leads by a  $3 \times 3$  scattering matrix  $S$ , where each diagonal element denotes the RC of the respective channel, and off-diagonal elements are the respective TCs [9,10]. In Fig. 1, channel 1 is chosen to be the one corresponding to the leads, and channels 2 and 3 correspond to the upper and lower arms, respectively, for both junctions. The current-conservation and time-reversal invariance imply that  $S$  is unitary and symmetric [11]. Geometric symmetry of the two channels 2 and 3 is assumed, which gives  $S_{12} = S_{13}$  and  $S_{22} = S_{33}$ . And  $S_{11} = 0$  is also assumed to correspond to a strong coupling between the leads and the ring [9–11].

In order to simulate the point contact, through which the ring is coupled to the large two-dimensional electron gas (2DEG) reservoir in the experiment [1], an ideal geometric point-junction is used. As a result, the continuity of the wave function gives rise to that the TCs,  $S_{ij}$  ( $i \neq j$ ) in the scattering matrix  $S$ , through the junctions are real and positive. Then, the scattering matrix  $S$  is obtained as

$$S = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \end{pmatrix}. \quad (1)$$

A more general discussion was done by Büttiker et al. in 1984 [11], where a general real matrix  $S$  was considered. However, the effect of the sign of the matrix elements has not been identified yet. Though the character of the whole system does not depend qualitatively on the choice of matrix  $S$  [9], and the total transmission probability is independent on the sign of the matrix elements in a symmetric system [11], it is no longer a trivial thing in an unsymmetric system. It is obvious that a negative

TC, in comparison with a positive TC, means an additional phase change of  $\pi$  for electrons transporting through the junction, which is not suitable for the description of the device in the experiment [1].

By using the scattering matrix given in Eq. (1), the total TC of the whole system can be expressed as

$$t = 2 \frac{t_1(1+r_2)(1+r'_2) + t_2(1+r_1)(1+r'_1) - t_1 t_2 (t'_1 + t'_2)}{(2+r_1+r_2)(2+r'_1+r'_2) - (t_1+t_2)(t'_1+t'_2)}. \quad (2)$$

In order to focus on the effects of the QD, we study a simple example of the perfect ring without scattering in the arms, which gives  $t_1 = t'_1 = t_D = |t_D|e^{i\alpha}$ ,  $r_1 = r_D = |r_D|e^{i\alpha}e^{i\beta}$ ,  $r'_1 = r'_D = -|r_D|e^{i\alpha}e^{-i\beta}$ ,  $t_2 = t'_2 = 1$ ,  $r_2 = r'_2 = 0$ . Here  $t_D$  ( $r_D$ ) and  $t'_D$  ( $r'_D$ ) are the TC (RC) from the left and the right side of the QD, respectively.  $\alpha$  is the transmission phase shift introduced by the QD while  $\beta$  represents the phase property of the RC from both sides. Moreover, when magnetic flux  $\Phi$  is applied through the ring, its effect can be introduced by a Peierls phase factor  $\varphi = 2\pi\Phi/\Phi_0$  of the TCs, which can be absorbed in the TCs as  $t_1 \rightarrow t_1 e^{-i\varphi/2}$ ,  $t'_1 \rightarrow t'_1 e^{i\varphi/2}$ ,  $t_2 \rightarrow t_2 e^{i\varphi/2}$ ,  $t'_2 \rightarrow t'_2 e^{-i\varphi/2}$ . Here  $\Phi_0$  is the elementary flux quantum  $\Phi_0 = hc/e$ . In this system, the total transmission coefficient  $t$  is given by

$$t = \frac{(\sin \alpha - |r_D| \sin \beta)}{(\sin \alpha - |r_D| \sin \beta) + \frac{1}{2}i(\cos \alpha - |t_D| \cos \varphi)} e^{i\varphi/2}, \quad (3)$$

and yields a transmission probability

$$T = |t|^2 = \frac{(\sin \alpha - |r_D| \sin \beta)^2}{(\sin \alpha - |r_D| \sin \beta)^2 + \frac{1}{4}(\cos \alpha - |t_D| \cos \varphi)^2}. \quad (4)$$

It is obvious that the dependence of the transmission probability, and thus, the conductance which is proportional to  $T$  according to Büttiker's formula, on the flux  $\Phi$  is no longer a simple cosine-relation, though it is still an even function of the flux [11]. In other words, the resonance of the system can not be described by the concept of a "phase shift" near the resonance of the QD [5], in which case one has

$|t_D| \approx 1$ . For the sake of comparison, however, we can still define the phase  $\theta$  of the conductance oscillation (CO) (which corresponds to the measured phase in the experiment [1]) as the  $\varphi$ , at which  $T$  firstly reaches a maximum with increasing  $\varphi$  from  $\varphi = 0$  [7].

Two kinds of situations exist in Eq. (4). For  $|t_D| \leq |\cos \alpha|$ ,  $T$  is a monotonous function of  $\varphi$  in the region  $\varphi \in [0, \pi]$ . Particularly,  $T$  reaches a maximum at  $\varphi = 0$  for  $\cos \alpha > 0$  and at  $\varphi = \pi$  for  $\cos \alpha < 0$  respectively. Thus one gets  $\theta = 0$  for  $\cos \alpha > 0$  and  $\theta = \pi$  for  $\cos \alpha < 0$  respectively. In this case, the phase difference between them can be defined normally, since the functional form of  $T$  is still cosine-like, though it is not a strict cosine-relation. For  $|t_D| > |\cos \alpha|$ , however,  $T$  is no longer a monotonous function of  $\varphi$  in the region  $\varphi \in [0, \pi]$ . Instead, there exist two maxima of  $T$ . In other words,  $T$  changes its functional form. Actually, this phenomena was indeed observed in the experiment [13]. In this case, the phase  $\theta$  defined by us is thus given by

$$\theta = \cos^{-1} [(\cos \alpha) / |t_D|], \quad (5)$$

which describes the transition behavior between the two extreme cases in the first situation. In general, this situation corresponds to the occurrence of a resonance through the QD.

In order to evaluate the effect of the QD on the transmission properties, we consider a wave transporting through the QD. The QD is modeled as a quantum well (well potential  $-V_G$ ), which corresponds to the plunger gate voltage on the QD [1], with two barriers (barrier heights  $V_B$ ) [6,7]. Here  $\epsilon = \hbar^2/2m^*a^2$  is taken to be the energy unit, where  $m^*$  is the effective mass of electrons, and the lattice constant  $a$  in the 2DEG is the length unit. By using the continuous condition of the wave function, the TCs and RCs of the QD can be obtained analytically. Moreover, an interesting relation is found between the TC and RC:

$$r_D = ikt_D, \quad (6)$$

where  $k$  is real. More detail analysis shows that this relation is due to  $r_D = r'_D$ , which is caused by the symmetry of the QD. Because  $\beta$  is defined as the phase difference between  $r_D$  and  $t_D$ , one then gets

$\beta = \pm \pi/2$ , with the sign the same as  $k$ . Moreover, the sign of  $k$  will be changed when the QD passes through a resonance with increasing  $V_G$ , which will result in that  $\beta$  changes by  $\pi$  from one side of the resonance to the other. It should be noted that this result is not qualitatively depended on the symmetry of the QD. In order to simplify the discussion, however, we restrict to the symmetric case.

Since  $\beta$  is equal to  $\pm \pi/2$ , the transmission properties of the QD are mainly characterized by the TC, which is calculated and shown in Fig. 2. It is well-known that each quasi-bound state of the QD will result in a resonant tunneling, namely a transmission amplitude peak, when the state is shifted to the Fermi energy by the variation of  $V_G$ . The finite width of the resonant peaks is caused by the finite height of the barrier potential  $V_B$ . Furthermore, with finite height of the barrier potential, the tunneling through the QD always results in the quantum coupling between the QD and the remain part of the whole system, and causes interactions between their quasi-bound states. The transmission phase  $\alpha$  introduced by the QD is shown as the thin line in Fig. 2. As what is expected, the phase changes by  $\pi$  as a resonant level of the QD is being scanned through the Fermi energy, while the sharp jump of  $2\pi$  is caused by the periodicity of the phase and means nothing. Moreover, the analytical result shows that the transmission phase is 0 or  $\pi$  alternatively at exact consequent resonance peaks ( $|t_D| = 1$ ), and is around  $\pi/2$  or  $-\pi/2$  when no resonance occurs.

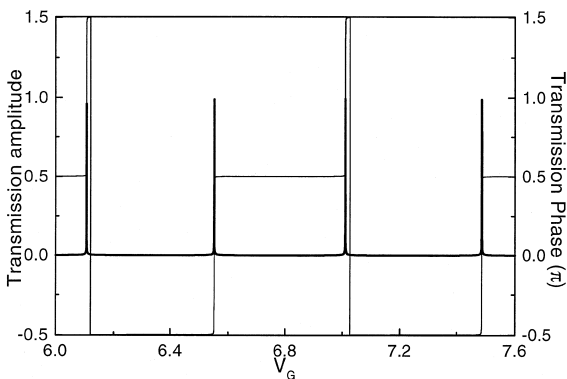


Fig. 2. The transmission amplitude  $|t_D|$  (solid line) and the transmission phase  $\alpha$  (thin line) of an isolated quantum dot as a function of the modeled well-potential  $V_G$ .

With the properties of the QD, then, let us consider the whole system under zero magnetic flux. In terms of  $t_D$  and  $r_D$ , Eq. (2) can be rewritten as

$$t = 2 \frac{1 + r_D + t_D}{3 + r_D + t_D}. \quad (7)$$

When no resonance occurs in the QD, one has  $|t_D| \approx 0$  and  $|r_D| \approx 1$ . Moreover, as a result of the full reflection, one has  $r_D \approx -1$ , which results in  $t \rightarrow 0$ . This result is in good agreement with the transmission property of a quantum waveguide with two stubs under long-wave-limit [12]. Actually, the QD with no resonance occurring will divide the upper arm of the ring into two stubs, and then, the whole system can be simulated by a quantum waveguide with two stubs. The waves reflected at the terminals of the stubs result in destructive interference, and reduce greatly the transmission through the whole system, though one branch of the interference device is a perfect waveguide. On the other hand, when a resonance occurs in the QD, one has  $|r_D| \approx 0$  and  $|t_D| \approx 1$ . It is also interesting to note that the phase  $\alpha$  of  $t_D$  will change alternatively by  $\pi$  for consequent resonant levels of the QD (as shown in Fig. 2). As mentioned above, the phase  $\alpha$  is either 0 or  $\pi$  at exact resonance peaks, which means one has exactly  $t_D = \pm 1$  at those points. This will result in two different total transmission probability  $T$  for two adjacent resonant levels of the QD:  $T \rightarrow 1$  for  $\alpha$  around 0 and  $T \rightarrow 0$  for  $\alpha$  around  $\pi$ . Obviously, it is caused by the interference effect. Noting the difference of phase shift introduced by the QD is  $\pi$  for two adjacent resonant levels, if one resonant level of the QD results in constructive interference between two arms of the ring, namely,  $T \rightarrow 1$ , the adjacent one will result in destructive interference, namely,  $T \rightarrow 0$ . The numerical results are shown in Fig. 3. For the sake of comparison, the numerical result under half flux quantum is also given in Fig. 3. In this case, the constructive and destructive interference effects are exchanged. It can also be easily concluded from Eq. (4) that the additional phase shift of  $\pi$  in  $\phi$  introduced by the magnetic flux alternates the constructive and destructive interference for adjacent resonant levels of the QD, which proves the conjecture in our previous simulative work [7].

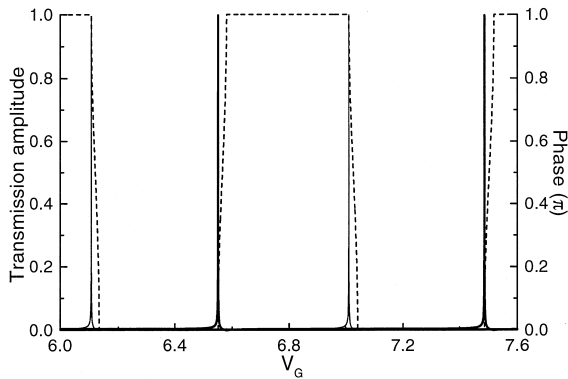


Fig. 3. Total transmission amplitude  $|t|$  as a function of the modeled well-potential  $V_G$  under zero (solid line) and half (thin line) magnetic flux quantum. The evaluated phase  $\theta$  of conductance oscillations as a function of the modeled well-potential  $V_G$  is given as dashed line.

The phase  $\theta$  of the CO is also evaluated, which is shown in Fig. 3. When the state of the QD is far from resonance,  $|\cos \alpha| > |t_D|$  is satisfied. As a result,  $\theta$  is either 0 or  $\pi$ . It should be mentioned that at each resonant peak of the QD one has  $|t_D| = 1$ , and  $\alpha$  is equal to 0 and  $\pi$  alternatively for adjacent peaks. Then, one obtains that the resonant peak position is just the critical point of the phase change (as shown in Fig. 3). Furthermore, in the resonant levels of the QD, which corresponds to the phase change region in our previous work [7], the phase  $\theta$  changes by  $\pi$  continuously. It can be seen from Eq. (5) that with finite height of the barrier, the phase  $\theta$  is a continuous function as a function of the well potential  $V_G$  in this region. And each jump of the phase corresponds to a resonant level of the QD. Numerical results also show that the width of the phase change region is determined by the barrier heights  $V_B$ . It has been pointed out that width of the phase change region is caused by the interactions between the quasi-bound states in the QD and those in the remain part in this system [7]. Obviously, higher barriers will reduce the quantum coupling between them and result in narrow phase change region.

Finally, let us check the phase of the TC through the whole system. When there is no resonance

through the QD, one has  $|t_D| \approx 0$ . It can be derived from Eq. (3) that the magnetic flux  $\Phi$  results in an additional phase shift of  $\varphi/2$  for the total transmission coefficient  $t$ , which also means that the arm without the QD dominates the phase of the total TC.

In summary, we have shown theoretically the transmission properties of a mesoscopic ring with a QD embedded in one of its arms within a one-dimensional scattering model. With the use of a scattering matrix describing the junctions between the leads and the ring, the effects of the quantum interference and the resonant tunneling through the QD are presented analytically. When the state of the dot is far from a resonance, the system acts as a quantum wire with two separated stubs at both ends. However, when a resonant tunneling through the dot occurs, an extra phase shift may be introduced to the wave through the dot and then the quantum interference effect may be flipped. Moreover, with the help of such an analytical result, the physical picture is shown clearly, which will surely support strongly our previous conclusions based on numerical calculations [7].

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